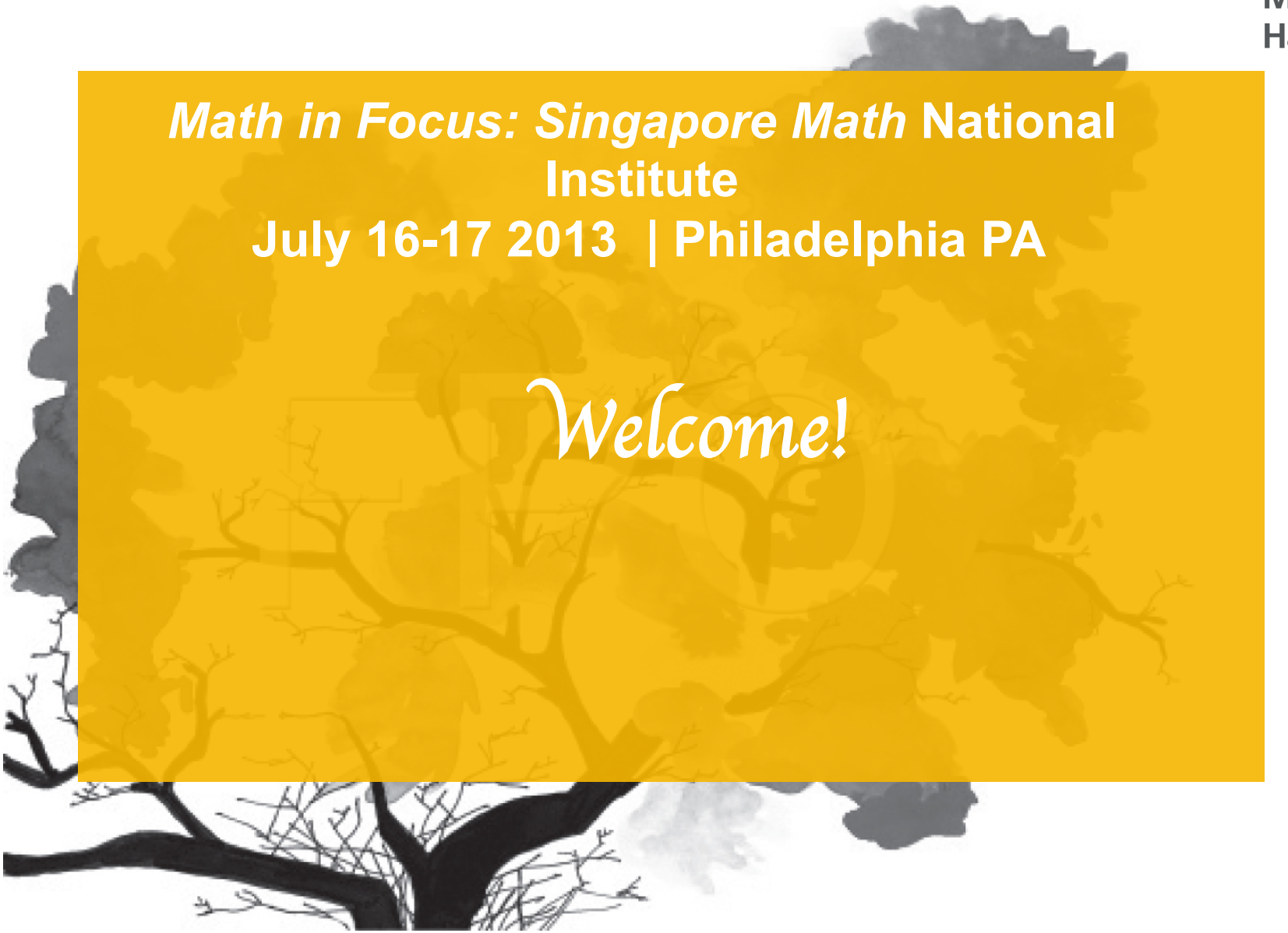
Math in Focus: Singapore Math National Institute

July 16-17 2013 | Philadelphia PA

Welcome!

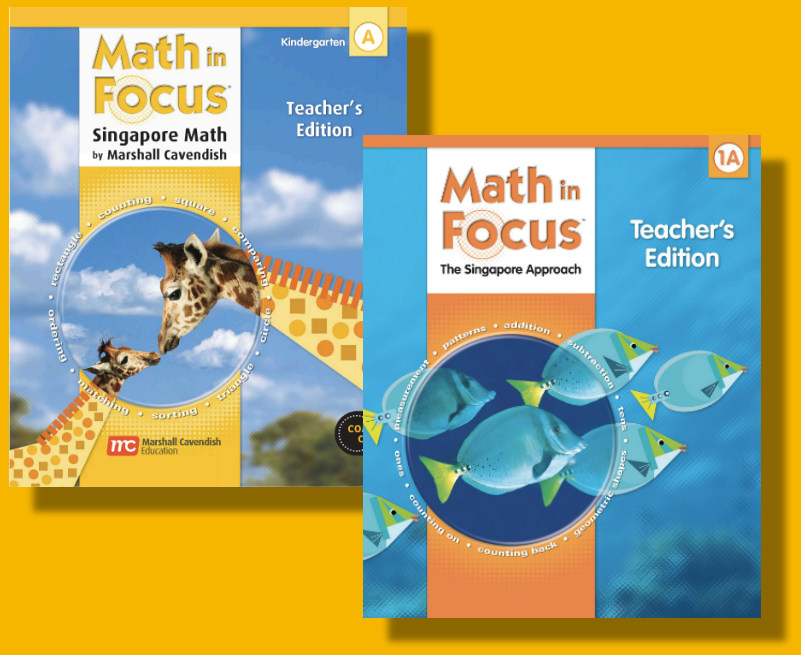


Beth Ardell

Houghton Mifflin Harcourt

Math in Focus

An In-Depth Look at Grades K-1



Big Ideas in Grades K and 1:

Mathematical Proficiency

Pacing in terms of importance

Understanding through assessing

"I am slow to learn and slow to forget that which I have learned.

My mind is like a piece of steel; very hard to scratch anything on it and almost impossible after you get it there to rub it out."

Abraham Lincoln
to his friend
Joshua Speed,
quoted in a letter
to *The New Yorker*

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What are the Pillars of *Math in Focus*?

What are the Pillars of *Math in Focus*?

- Concrete – Pictorial – Abstract Pedagogy
- Visualization with Variation
- Mathematics as Thinking

What do you know for sure about C-P-A?

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What do you know for sure about C-P-A?

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During Concrete experiences, students create authentic, meaningful mathematical situations.

At the Pictorial level, students have opportunities to reflect on their thinking and see their learning in different formats as they draw their own pictures and analyze pictures from the Student Book.

At the Abstract level, students represent the mathematics with symbols in a variety of ways.

What do you know for sure about Visualization with Variation?

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What do you know for sure about Math as Thinking?

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Frame for our work: Theory and Practice

Let's Discuss:

- Important Content and How it Develops

  Learn portion of the lesson is not to be taken lightly…This is the Anchor Task

Number and Operations

Kindergarten Content that should dig deep

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Important Content in Grades K and 1

- Composing and Decomposing Numbers in Kindergarten & First Grade
- Place Value Experiences in Number and Operations
- The “Everyday Language” of Mathematics

Why Number as the topic for Digging Deeper?

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Number Sense – a comfort with numbers and a well-developed understanding of place value – is one of the overarching goals of mathematics learning.

What else can you tell me about these numbers? How else can we express that number?

Stephen Leinwand

Common Core State Standards
What does CCSS have to say?

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Common Core - Kindergarten

Houghton Mifflin Harcourt

K.CC.5

Count to answer "how many?" questions about as many as 20 things arranged in a line, rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1 – 20, count out than many objects.

Teaching Number

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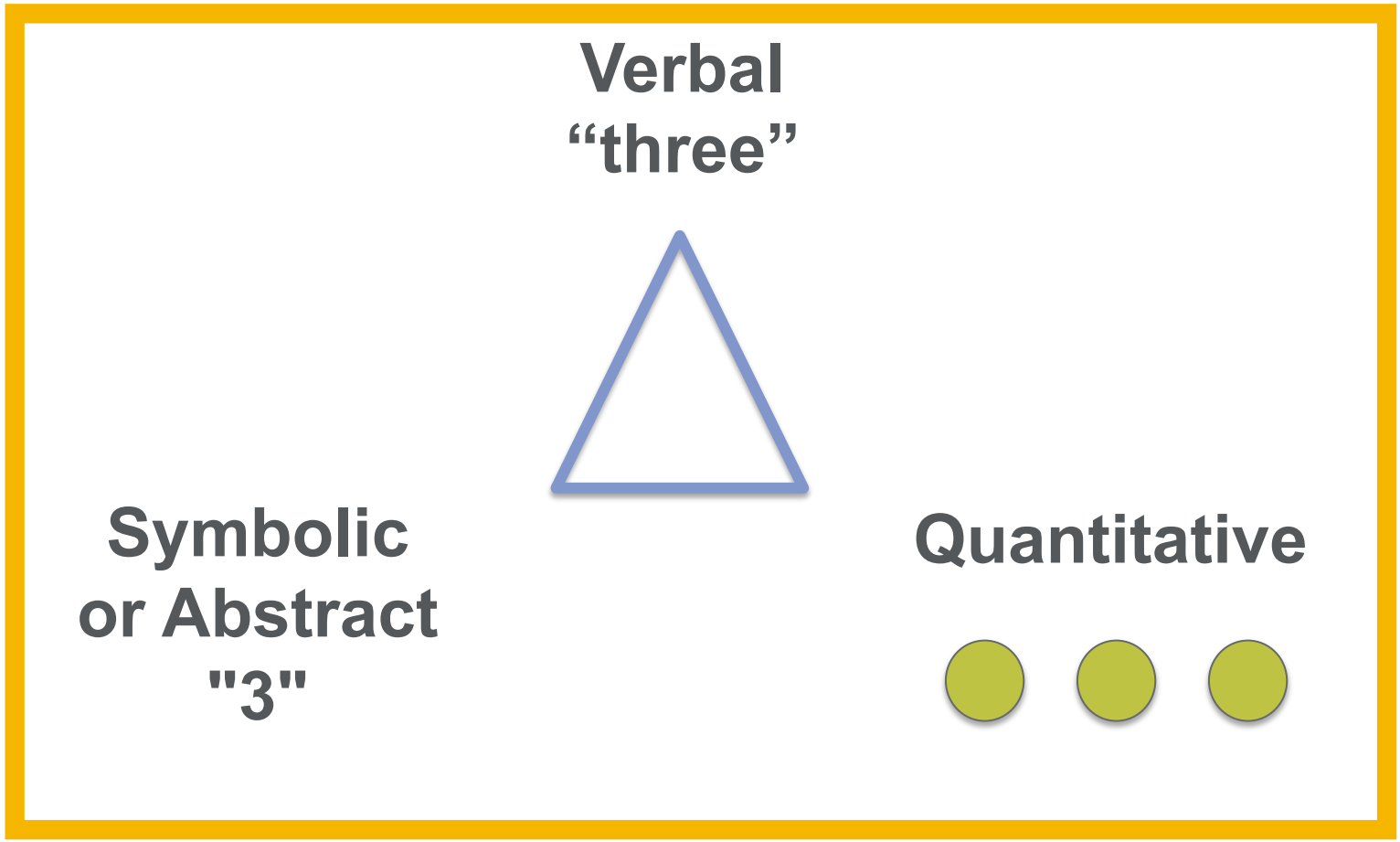
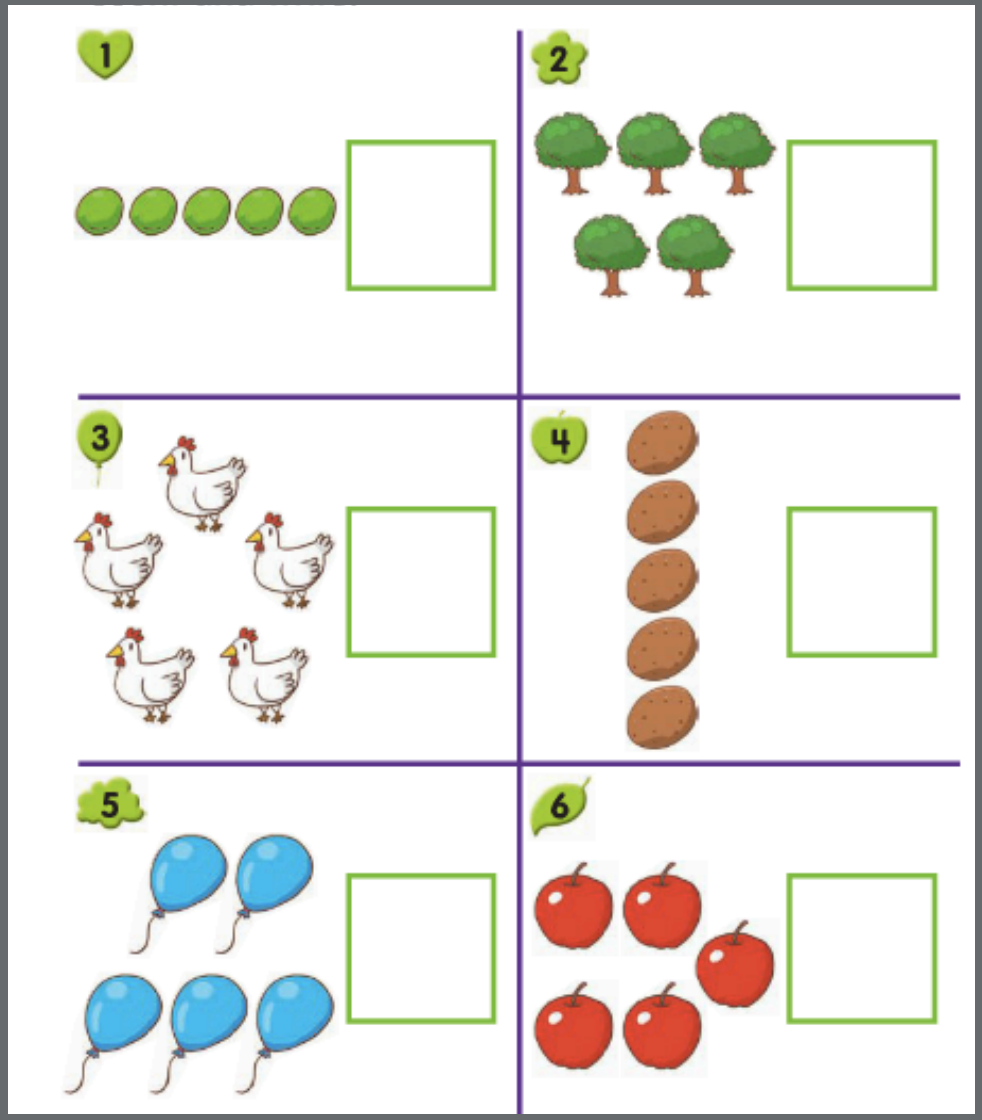


Kindergarten

- How would you use this Apply page from the Student Book?
- What is the obvious question?
- What are the real questions?



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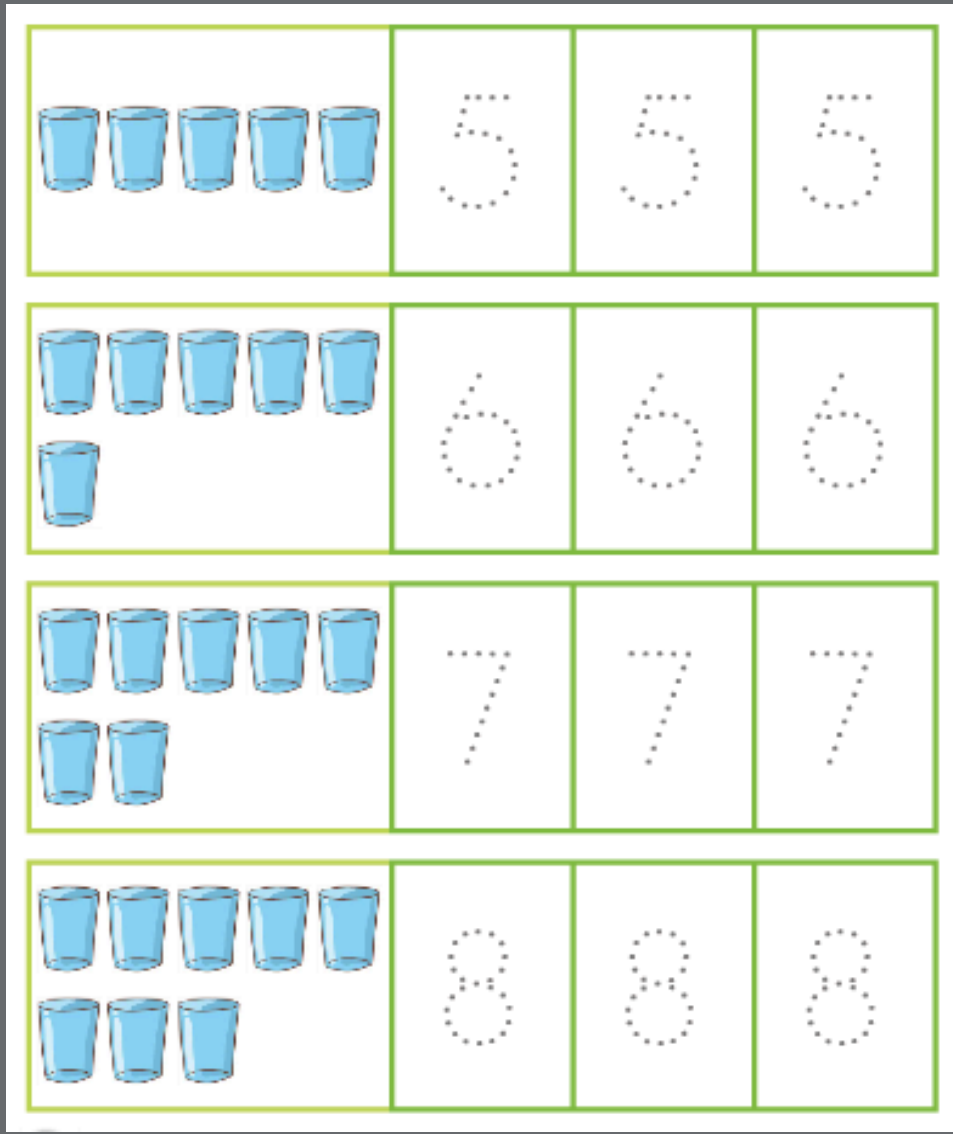
Kindergarten

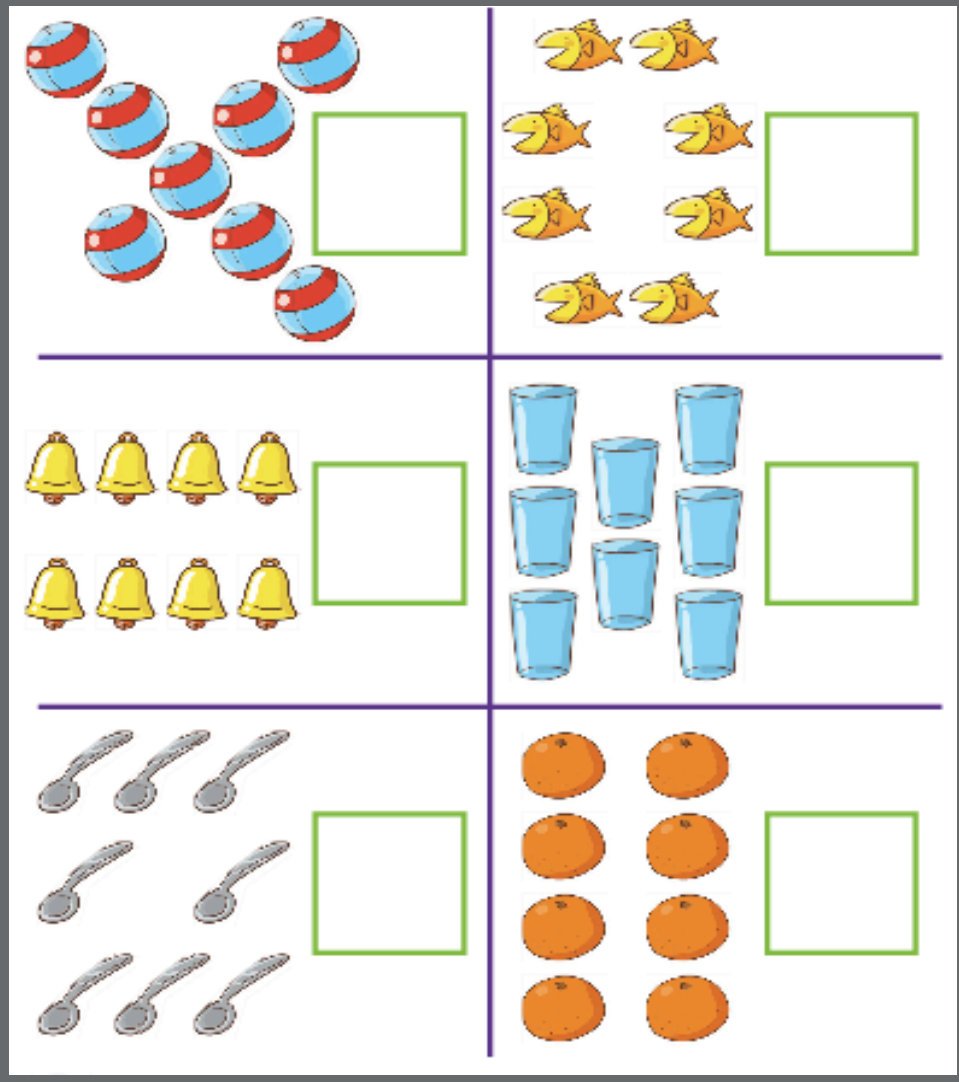
- What about this page?



Kindergarten

With a partner:

- Choose one of the arrangements to copy.
- Share why you chose it and how you might count it.
- Build four arrangements of your own.
- Discuss what you might observe.



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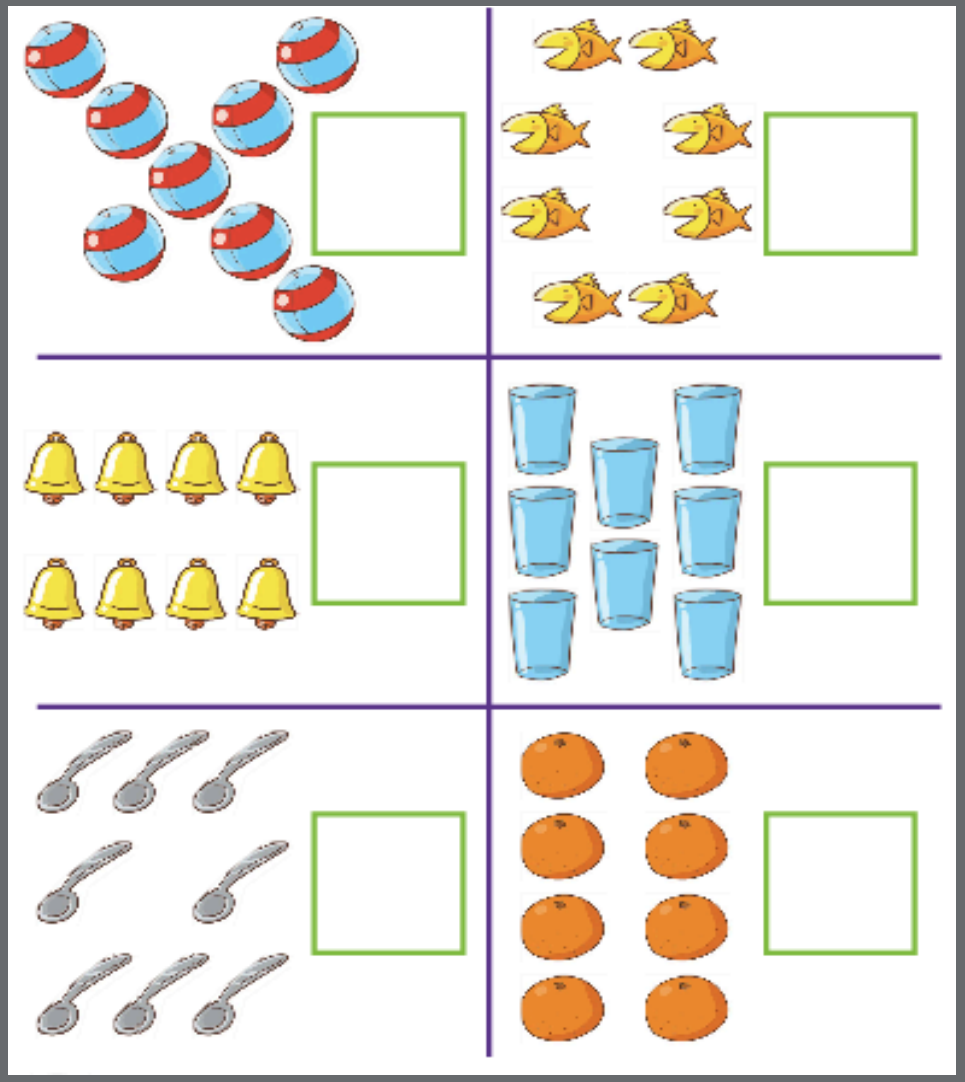
Kindergarten

- How do you see these quantities?



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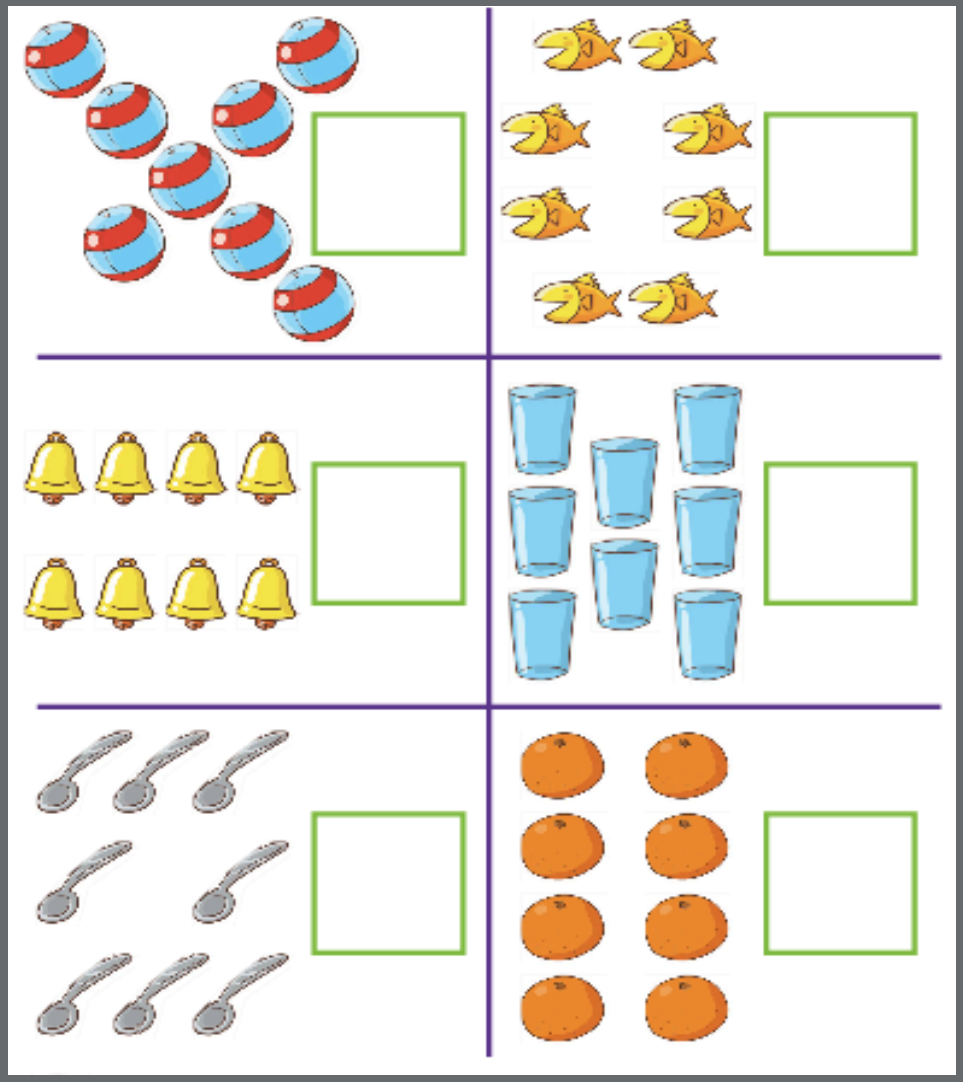
Kindergarten

- Why were these arrangements chosen?



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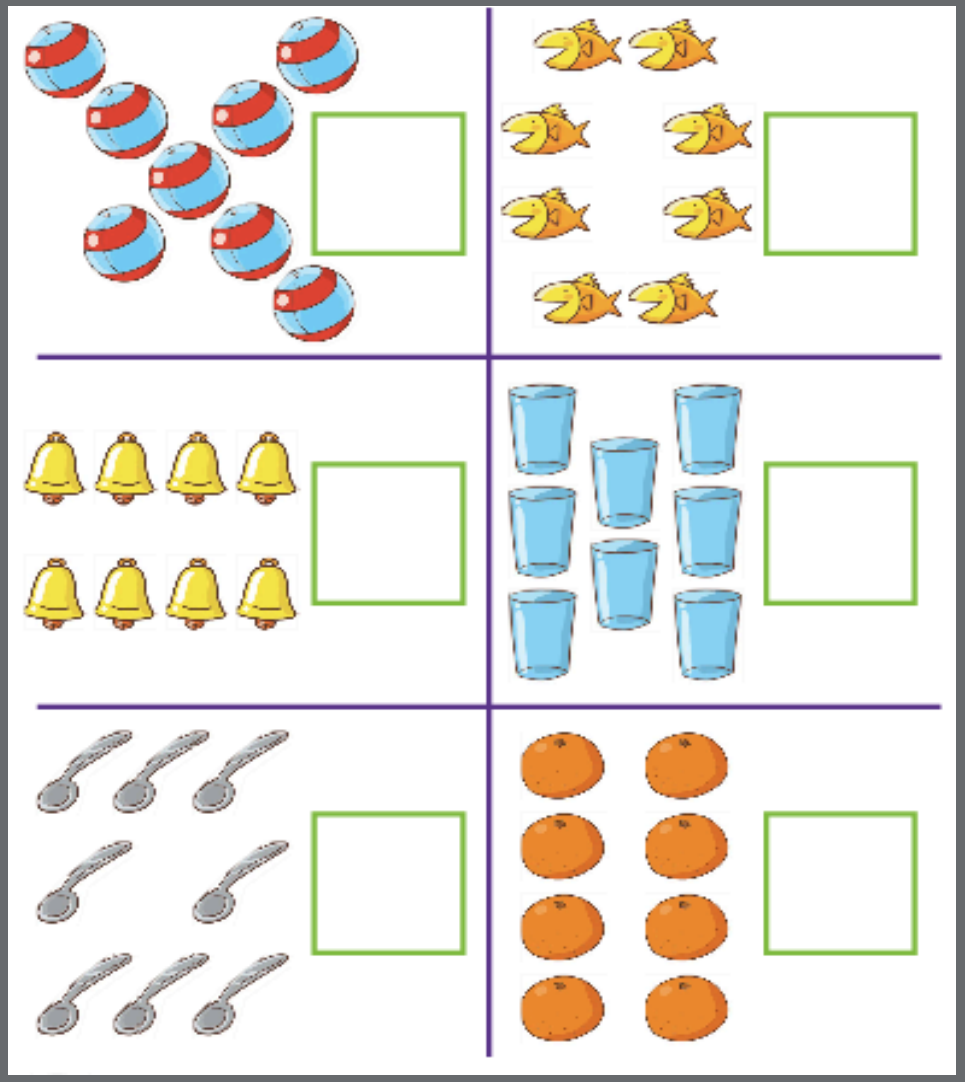
Kindergarten

- How do you "encourage" children to see that the same number of objects can be arranged in different ways?



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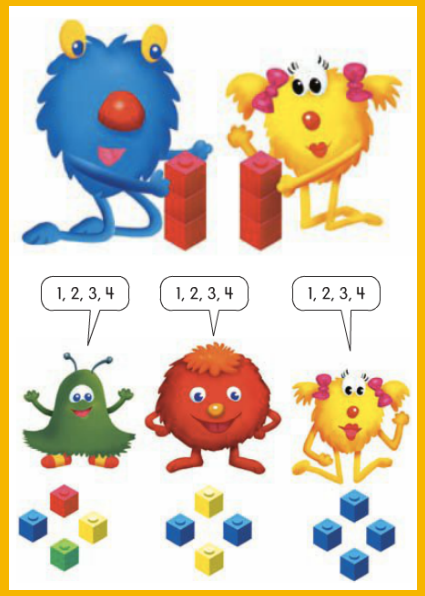
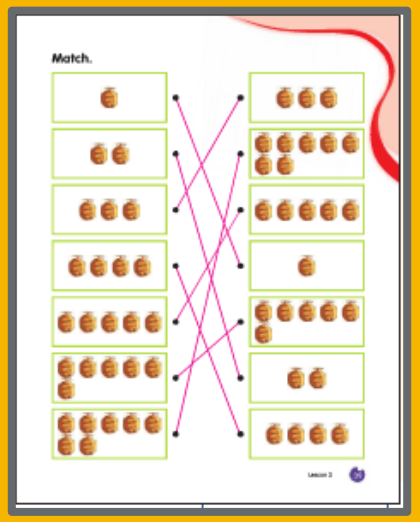
What is the Sequence for Teaching Number?

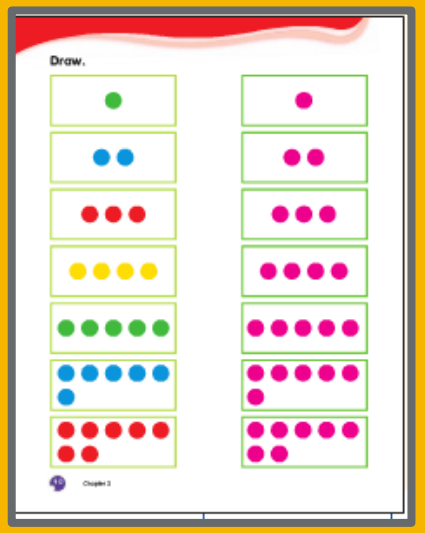
Build - Concrete



Match

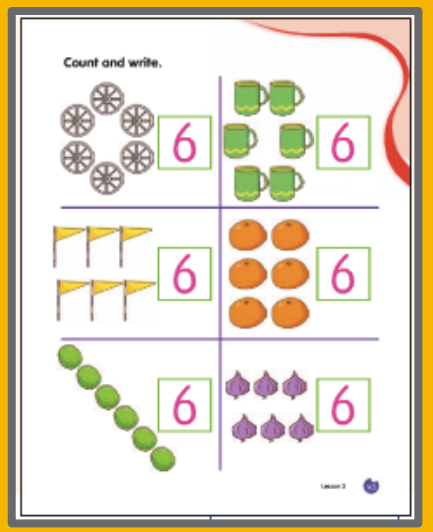


Draw



Trace

Count & Write



Common Core - Kindergarten

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K.OA.3

Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

Douglas Clements

A part – whole concept may be the foundation for understanding more advanced concepts of number such as place-value representation.

What does this look like in more advanced topics?

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Douglas Clements

A part – whole concept may be the foundation for understanding more advanced concepts of number such as place-value representation.

What might 123 look like with base ten blocks?

Douglas Clements

A part – whole concept may be the foundation for understanding more advanced concepts of number such as place-value representation.

The whole 123 can be decomposed into the parts of 1 hundred, 2 tens and 3 ones or 12 tens and 3 ones, or 1 hundred, 1 ten and 13 ones.

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Douglas Clements

A part – whole concept is considered to
be a conceptual basis for understanding
and solving missing-part word problems.

- ? + 3 = 5
- ? – 2 = 7
- ? and ? make 5

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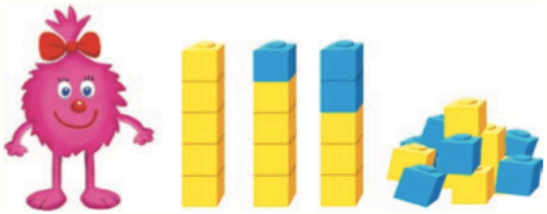
Kindergarten – Composing and Decomposing Numbers

- How many do we need to complete the tower?



ACTIVITY 3

Explore

Math Focus: Extend the concept of how numbers can be composed of other lesser numbers.

Materials: Connecting cubes, 30 per group and 30 for teacher (15 yellow and 15 blue)

Classroom Setup: Children work in small groups with teacher direction.

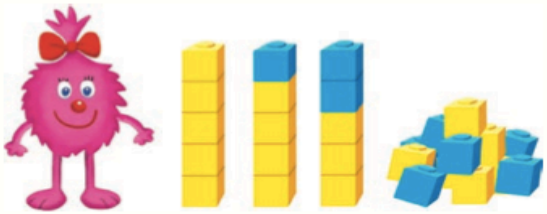
1. **Begin** the day by ***inviting*** children to gather around a table.
2. ***Model*** the activity to the children.
3. ***Tell*** children that you are going to build towers of 5 connecting cubes, but by using different combinations of cubes.
4. ***Start*** off by building a tower of 5 yellow cubes.
5. ***Count*** out the cubes. ***Ask:***
   - Do I need any blue cubes to complete this tower? (No)
   - How many blue cubes do I need to complete this tower? (0)
6. ***Say:*** 5 yellow cubes and 0 blue cubes make 5 cubes.
7. ***Place*** the tower aside.
8. Then, build a tower of 4 yellow cubes.
9. ***Count*** out the cubes. ***Ask:*** How many blue cubes do I need to complete this tower of 5? (1)
10. ***Fix*** on the 1 blue cube. Say: 4 yellow cubes and 1 blue cube make 5 cubes.
11. ***Place*** the tower aside.
12. ***Repeat*** steps 8 to 11 using the following combinations:
    - 3 yellow cubes and 2 blue cubes
    - 2 yellow cubes and 3 blue cubes
    - 1 yellow cube and 4 blue cubes
    - 0 yellow cubes and 5 blue cubes

Best Practices Do not alternate the colors of the cubes in the tower as this will make it difficult for children to see two distinct sets of cubes within the same tower.

13. ***Distribute*** materials to the children.
14. ***Ask*** them to do the same activity for towers of 4 cubes, 3 cubes and 2 cubes.
15. While children engage in the activity, **end** the day by asking check questions such as:
    - What were some ways you made up 4?
    - What were some ways you made up 2?

Kindergarten – Composing and Decomposing Numbers

- How might we dig deeper into this lesson?



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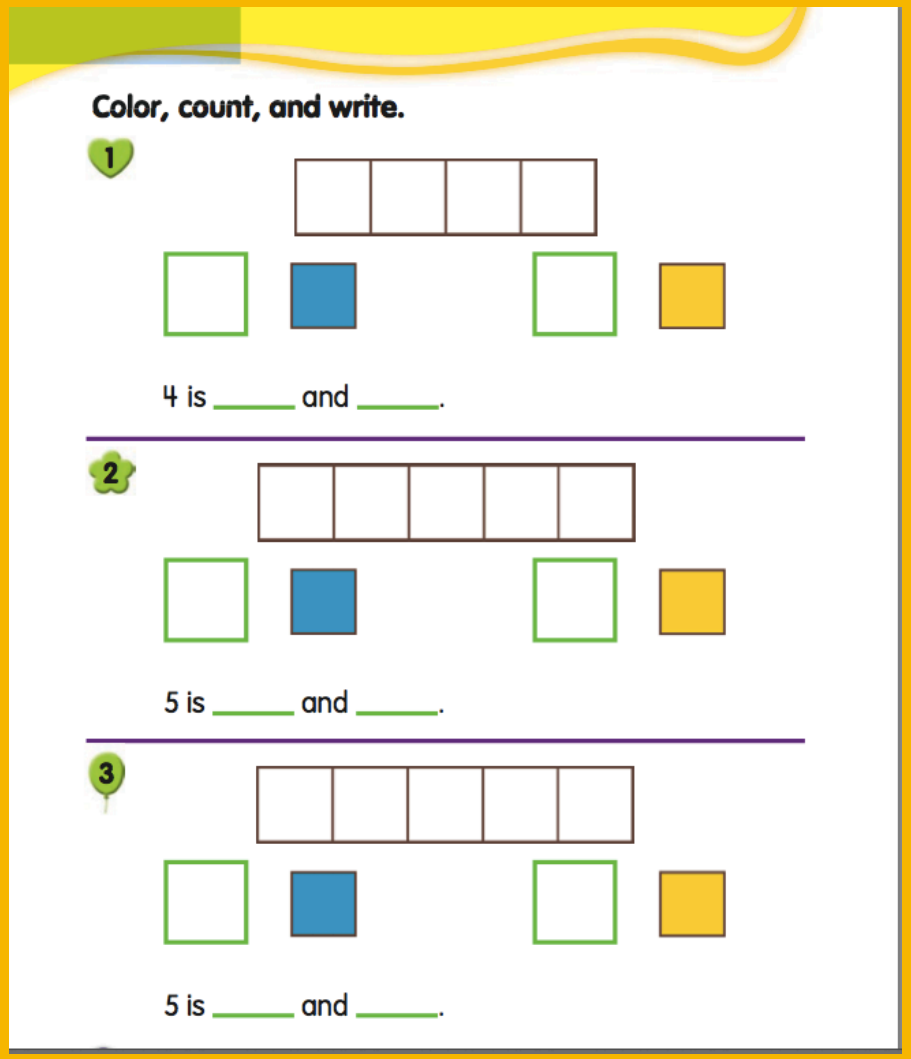
Kindergarten – Composing and Decomposing Numbers

What challenges might you anticipate for this Apply page?



Number (Place Value) and Operations

Kindergarten Content that should dig deep

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Common Core - Kindergarten

Houghton Mifflin Harcourt

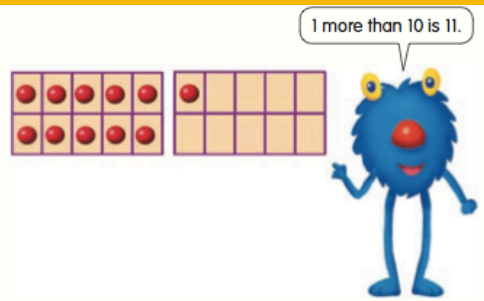
K.NBT.1

Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

With a partner:

- Build any number between 11 and 20.
- What are the real questions?
- What understanding are you looking for?

Kindergarten – Numbers to 20 – Chapter 6



ACTIVITY 3

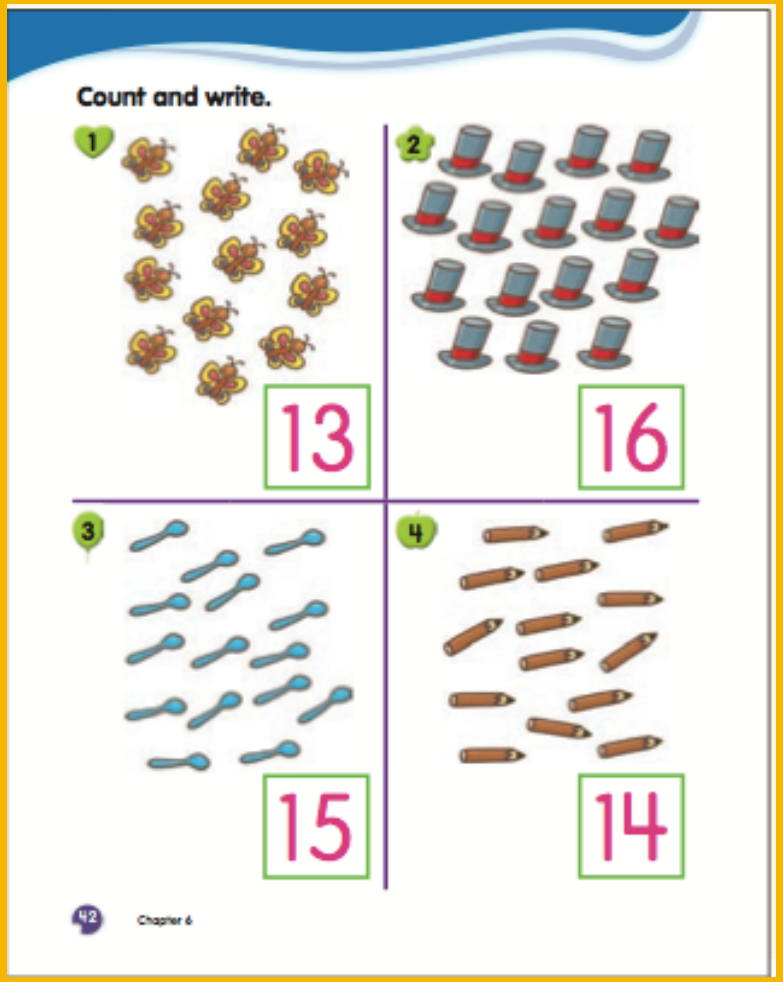
Explore

Math Focus: Extend the concept of 12.
Materials: Ten-frame (TR19), 1 copy per pair
Counters, 15 per pair
Classroom Setup: Children work in pairs at the math center.

1. **Begin** the day by ***asking*** children to gather around a table.
2. ***Invite*** a volunteer to help you model the activity.
3. ***Show*** children the ten-frame.
4. ***Explain*** that it is called a ten-frame because it has ten boxes.
5. ***Tell*** children that when you fill up the ten-frame, this means that it contains ten counters.
6. ***Ask*** the volunteer to hand you 10–12 counters. He or she is to quietly count the counters first and then give them to you without telling you how many there are. For example, you get 11 counters.
7. ***Fill*** up the ten-frame. ***Say:*** I have 10 counters here. And I have 1 more here. 1 more than 10 is 11. I have 11 counters.
8. Have the volunteer confirm this.
9. ***Count*** with the children. Point at the counters and ***say:*** 10, 11.
10. ***Repeat*** the activity with 12 counters. Point and ***say:*** 10, 11, 12.

Best Practices If the initial number of counters was 12, step 10 would illustrate counting up to 11.

11. ***Distribute*** materials to the children.
12. After modeling the activity, let children work in pairs at the math center.
13. Children take turns to play the roles of the teacher and the volunteer. Ensure that children exchange roles.
14. While children engage in the activity, ask check questions such as:
    - What will my ten-frame look like if I was accidentally given 9 counters?
    - How do we know 12 is greater that 11?
    - If we have a full ten-frame, how many do we have? (10)
    - What does 11 look like on the ten-frames? (A full frame and one more)
    - What does 12 look like on the ten-frames? (A full frame and two more)



Kindergarten – Numbers to 100 – Chapter 8

ACTIVITY 3

Explore

Math Focus: Extend the concept of sequencing numbers 20 to 49.

Materials: 20-49 Numeral Cards (TR32a–c), 1 set per group
Connecting cubes, 50 per group

Classroom Setup: Children work in small groups at the math center.

1. **Begin** the day by ***inviting*** children to gather around a table.
2. ***Invite*** three volunteers to help you model the activity.
3. ***Give*** Child A the numeral cards and Child B the connecting cubes.
4. ***Tell*** Child A to choose a numeral card. For example, he or she chooses '25'.
5. Child A then gives this card to Child B and the rest of the cards to Child C.
6. ***Explain*** to Child B that he or she is to build a collection of cubes (made up of towers and loose cubes) according to the numeral card. And then build towers of the numbers that come just before and just after that number. In this case, Child B will build collections for 24, 25, and 26.
7. ***Explain*** to Child C that he or she is to pick the numeral cards to match Child B's 'before' and 'after' collection of cubes.
8. Child B then builds two 10-cubed towers and 5 single cubes to illustrate '25'.
9. ***Ask:*** Is this correct? (Yes)
10. Child B then continues building the cube collections for 24 and 26.

Best Practices Ensure that Child B displays 10-cubed towers and loose cubes to display each number.

11. After Child B has finished building the cube collections for 24, 25, and 26, Child C displays the numeral cards for the 24- and 26-cube collections.
12. ***Ask:***
    - Is this correct?
    - Can you read out the numbers for me?
13. ***Distribute*** materials to the children.

Common Core – Kindergarten

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K.OA.4

For any number 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

How will you facilitate this understanding for children? When was this "big idea" first introduced?

Kindergarten – Working with Ten (Chapters 12 & 14)

- How many more to make ten?
- How did your mind figure this out? How can you show your thinking with cubes?
- What language can we use to “coach” our students with this big idea?

Where was the Visualization?

Where was the Variation?

What Thinking did you experience?

What Mathematical Proficiencies were you looking for?

Number and Operations

Grade 1 Content that should dig deep

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Common Core State Standards
What does CCSS have to say?

Houghton Mifflin Harcourt

Common Core – Grade 1

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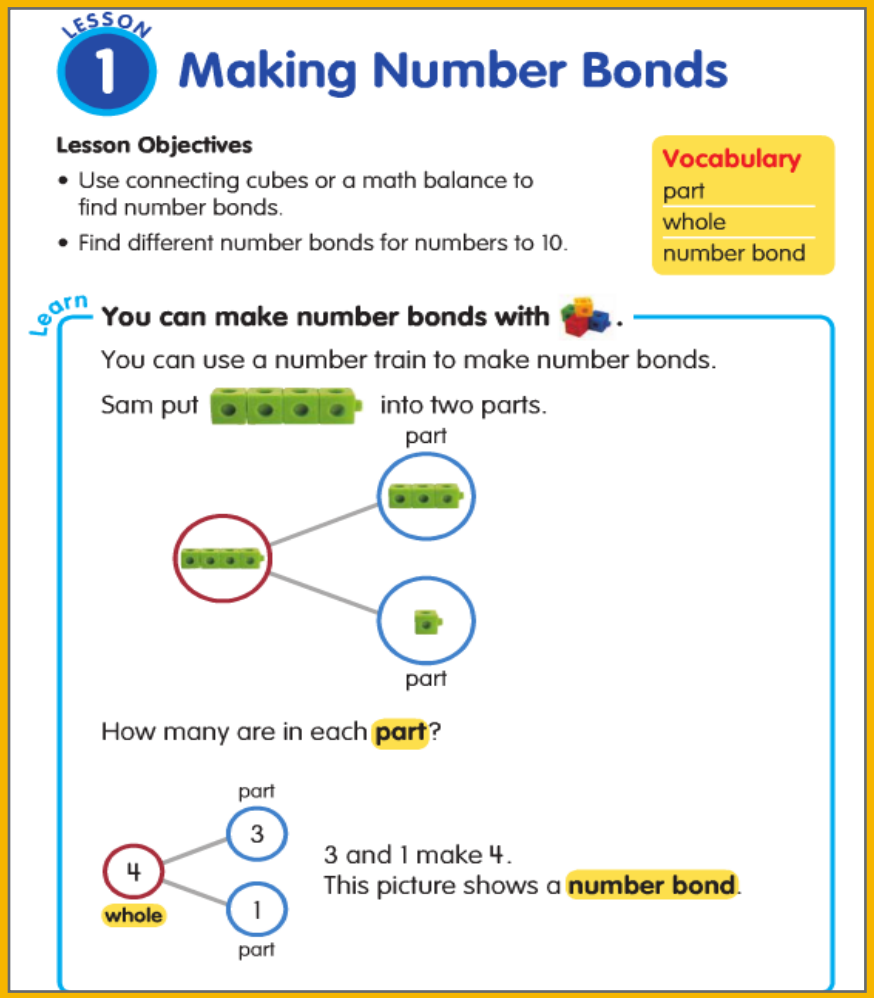
1.OA.6

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten, decomposing a number leading to a ten, using the relationship between addition and subtraction, and creating equivalent but easier or known sums.

Grade 1 – Composing and Decomposing Numbers (Ch. 2)

- Number Bonds explore the parts of the whole
- What questions might you ask?
- How can you differentiate in your use of number bonds?



LESSON 1

Making Number Bonds

Lesson Objectives

- Use connecting cubes or a math balance to find number bonds.
- Find different number bonds for numbers to 10.

Vocabulary

part
whole
number bond

Learn

You can make number bonds with .

You can use a number train to make number bonds.

Sam put into two parts.

part

part

How many are in each **part**?

part

3

4

whole

1

part

3 and 1 make 4.

This picture shows a **number bond**.

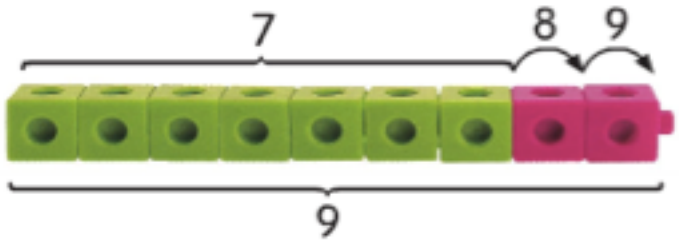
1st Grade

Addition Facts to 10 (Ch. 3)

Learn

You can count on to find how many more.

What is 2 **more than** 7?



More than means **added on to.**

7, 8, 9

What are the "Big Ideas" in this chapter?

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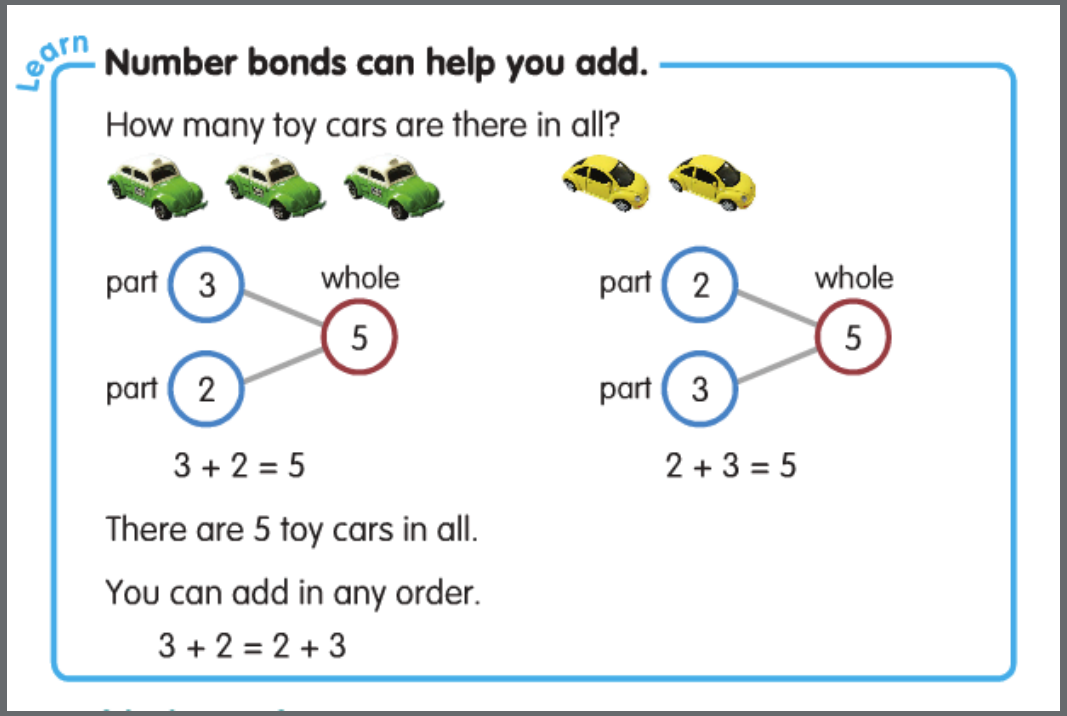
1st Grade

Addition Facts to 10 (Ch. 3)



What does pacing look like?

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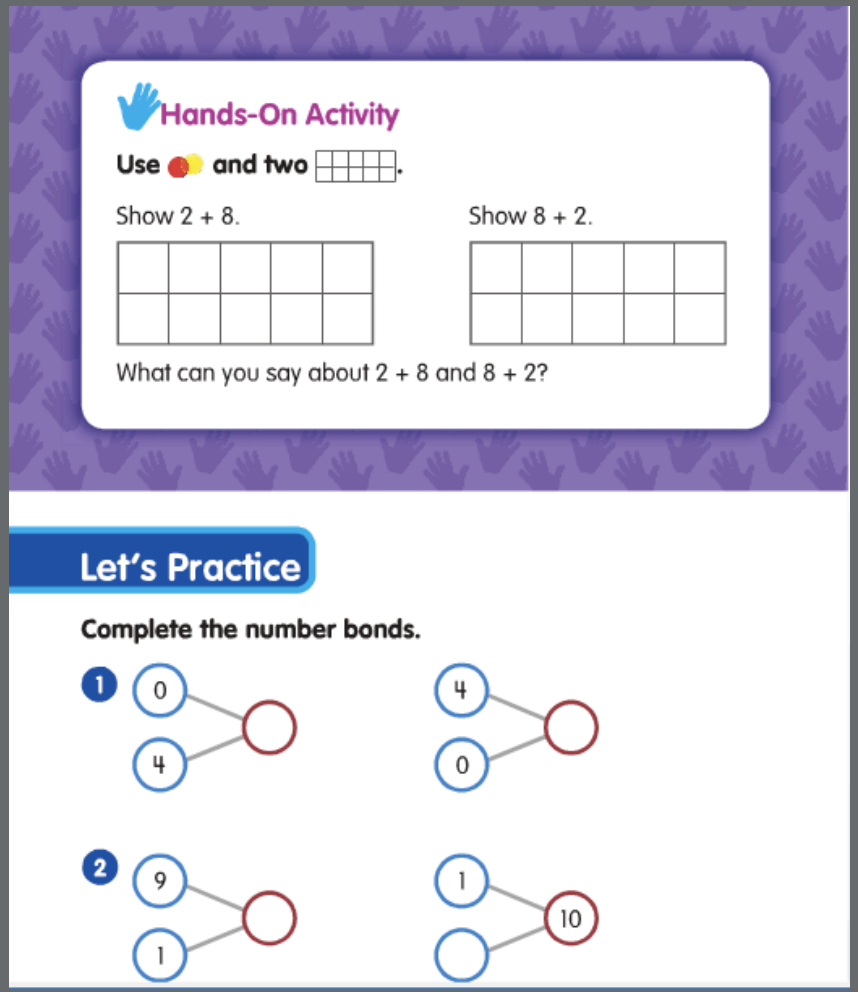
1st Grade

Addition Facts to 10 (Ch. 3)



Hands-On Activity

Use ● and two ▦.

Show 2 + 8.

Show 8 + 2.

What can you say about 2 + 8 and 8 + 2?

Let's Practice

Complete the number bonds.

1. 0, 4 — 4, 0

2. 9, 1 — 1, 10

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1st Grade

Addition Facts to 10 (Ch. 3)

You can tell addition stories about a picture.

What might this lesson look like in your classroom?

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1st Grade

Addition Facts to 10 (Ch. 3)

Tell an addition story about the
birds
turtles
bikes

Show with a number sentence and a number bond.

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1st Grade

Addition Facts to 10 (Ch. 3)

You can tell addition stories about a picture.

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1st Grade

Subtraction Facts to 10 (Ch. 3)

You can subtract by taking away.

What are “Big Ideas” in this chapter?

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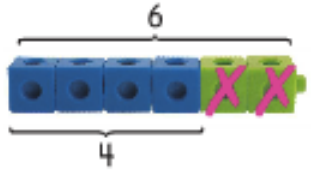
1st Grade

Subtraction Facts to 10 (Ch. 3)

Learn

You can take away to find how many less.

What is 2 less than 6?



6 – 2 = 4

4 is 2 less than 6.

Less than means **taken away from.**

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1st Grade

Subtraction Facts to 10 (Ch. 3)

Find 9 – 6.
Count on from the number that is less, 6.
Stop at 9.

3 steps

9 – 6 = 3

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1st Grade

Subtraction Facts to 10 (Ch. 3)

How do we help children to access the idea of Counting Up?

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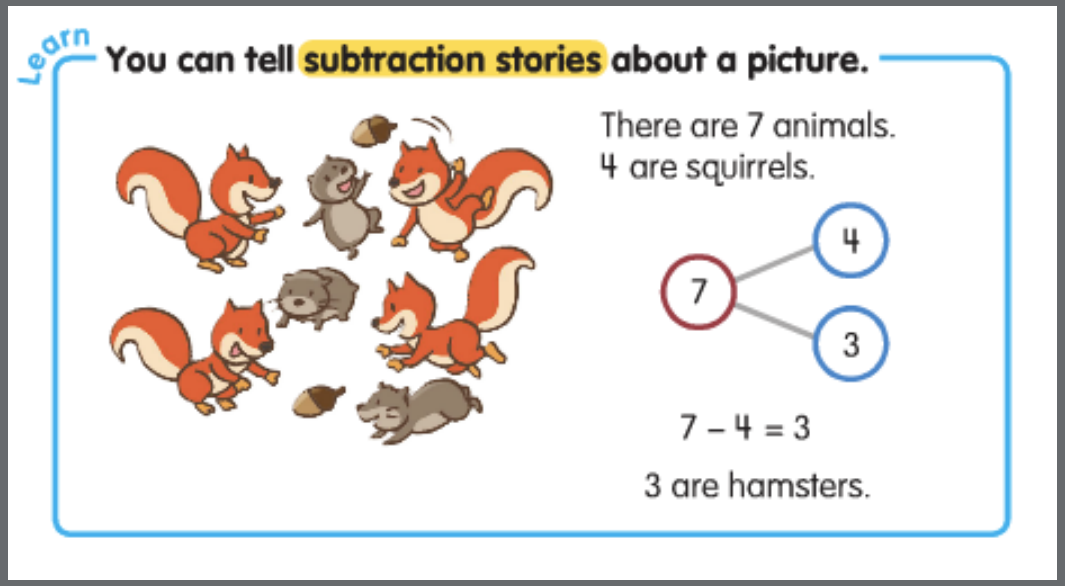
1st Grade

Subtraction Facts to 10 (Ch. 3)



How do we help children to access the idea of a Missing Part?

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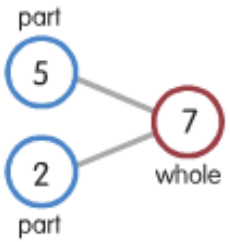
1st Grade

Subtraction to 10 (Ch. 3)

When do children really know Fact Families?

Learn

Addition and subtraction are related.



How many balls of wool are yellow?

$7 - 2 = 5$

How many balls of wool are blue?

$7 - 5 = 2$

How many balls of wool are there in all?

$2 + 5 = 7$ or $5 + 2 = 7$

$7 - 2 = 5$ $\quad 7 - 5 = 2$ $\quad 2 + 5 = 7$ $\quad 5 + 2 = 7$

This is a **fact family**.

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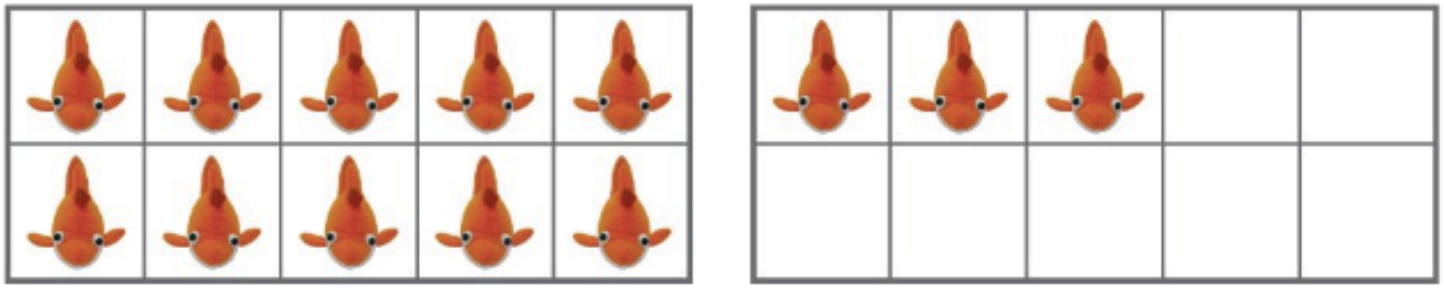
1st Grade

Numbers to 20 (Ch. 7)

Learn

You can first make a ten. Then count on.



10 and 3 make 13.
Ten and three make thirteen.
10 + 3 = 13

What are "Big Ideas" in this chapter?

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1st Grade

Addition to 20

Learn

You can add by making a 10.

Gus has 8 cherries.
Ava gives him 6 more.

How many cherries does Gus have now?

How would you connect this thinking to a number bond?

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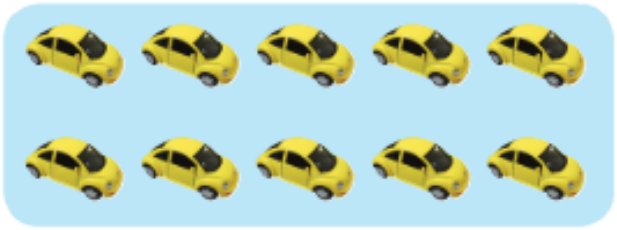
1st Grade

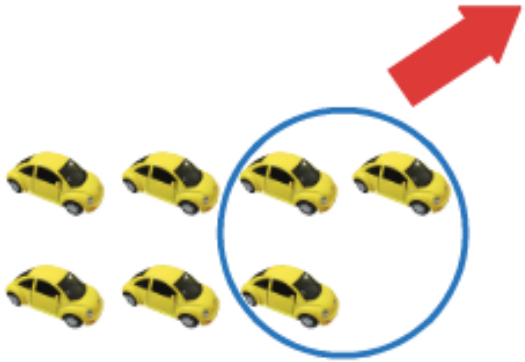
Subtraction to 20

Learn

You can subtract by grouping into a 10 and ones.

Ray has 17 toy cars.
He gives away 3 toy cars.
How many cars does he have left?





How would you launch this Anchor Task?
Build 17 on your double ten frame. Discuss how this problem could unfold in the classroom?

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1st Grade

Subtraction to 20

Learn

You can subtract by grouping into a 10 and ones.

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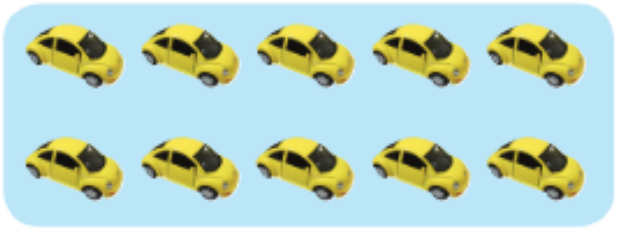

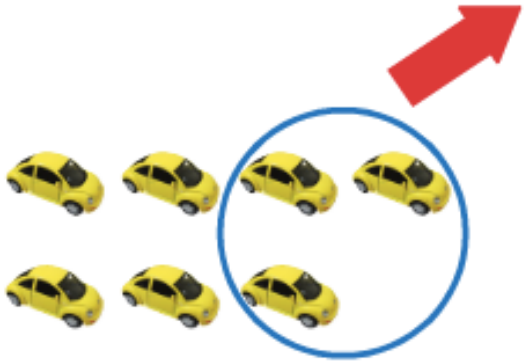

How would you launch this Anchor Task?
Build 17 on your double ten frame. Discuss how this problem could unfold in the classroom?

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1st Grade

Subtraction to 20

Learn

You can subtract by grouping into a 10 and ones.

Shawn makes 12 stars.
He gives 7 to Gina.
How many stars does Shawn have left?

What other problems might you use for practice?

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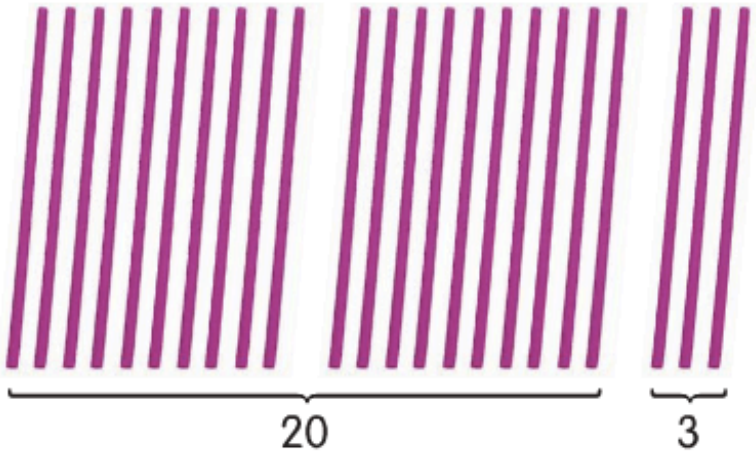
1st Grade

Numbers to 40

Learn

You can use place value to show numbers to 40.



Tens	Ones
2	3

23 = 2 tens 3 ones

23 = 20 + 3

Notice the representations

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Common Core – Grade 1

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1.NBT.4

Add within 100, including adding a two-digit number, and adding a two-digit number and multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations and / or the relations between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers one adds ten and tens, one and ones and sometimes it is necessary to compose a ten.

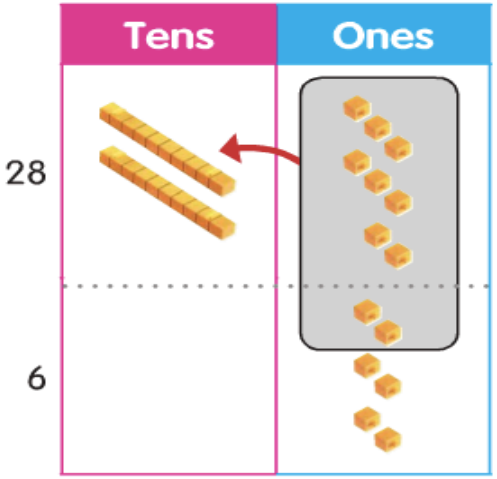
1st Grade

Addition and Subtraction to 40

Learn

You can use place-value charts to add ones to a number with regrouping.

28 + 6 = ?



Step 1 Add the ones.

	Tens	Ones
	1	
	2	8
+		6
	3	4

8 ones + 6 ones = 14 ones

Regroup the ones.

14 ones = 1 ten 4 ones

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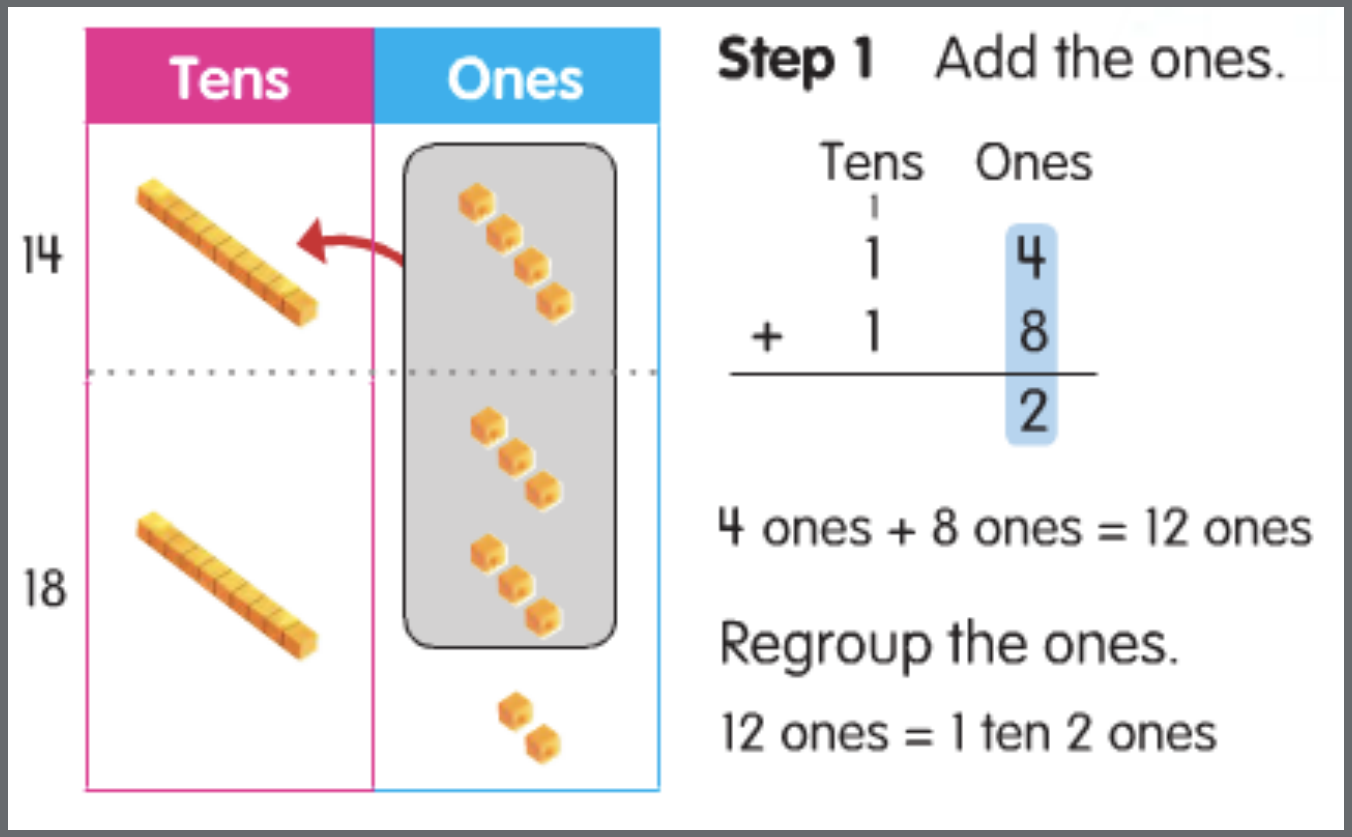
1st Grade

Addition and Subtraction to 40



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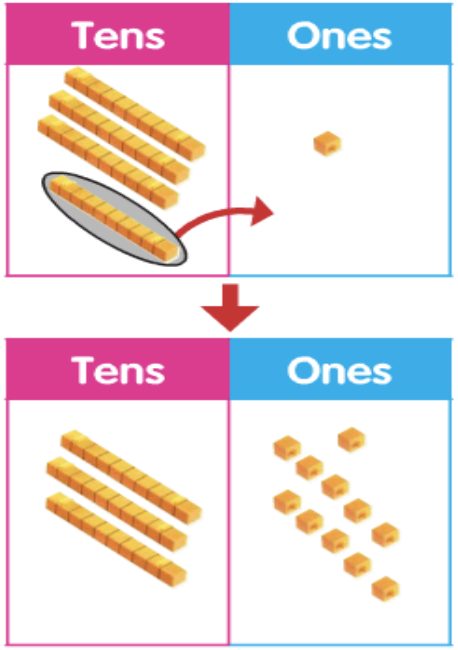
1st Grade

Addition and Subtraction to 40

Learn

You can use place-value charts to subtract numbers with regrouping.

41 – 29 = ?



Step 1 Subtract the ones.

You cannot subtract 9 ones from 1 one! So, you need to regroup.

Regroup the tens and ones in 41.

41 = 4 tens 1 one
= 3 tens 11 ones

Subtract.

	Tens	Ones
	3 4	11 1
–	2	9
		2

11 ones – 9 ones = 2 ones

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Where was the Visualization?

Where was the Variation?

What Thinking did you experience?

What Mathematical Proficiencies were you looking for?

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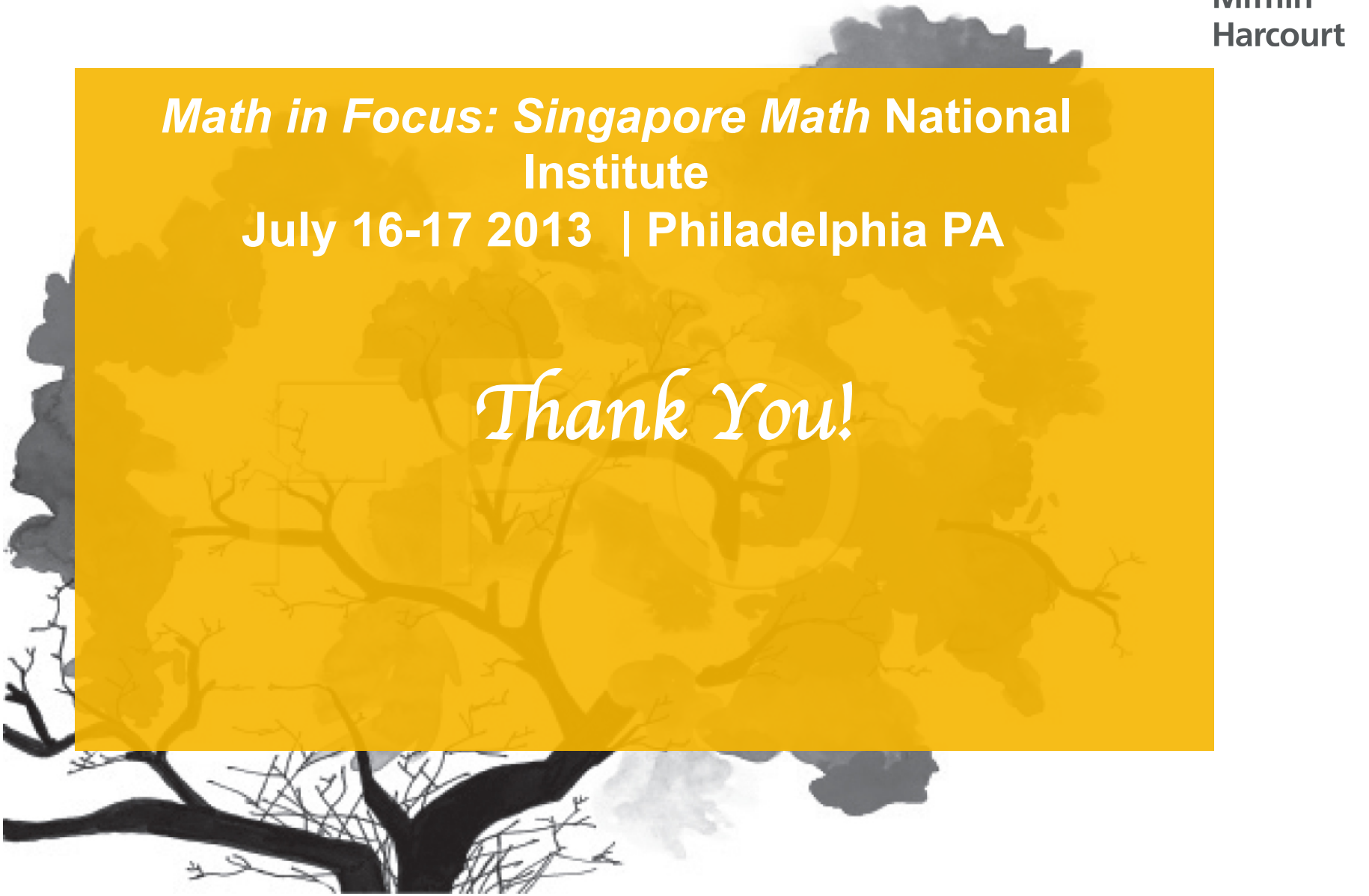

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Thank You!